Chapter 1: Numerical Expressions and Factors

GCF and LCM

1.4 Lesson

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Because 2 is factor of 10 and $2 \cdot 5 = 10$, 5 is also a factor of 10. The pair 2, 5 is called a **factor pair** of 10.

EXAMPLE 1 Finding Factor Pairs

The brass section of a marching band has 30 members. The band director arranges the brass section in rows. Each row has the same number of members. How many possible arrangements are there?

Use the factor pairs of 30 to find the number of arrangements.

$30 = 1 \cdot 30$	There could be 1 row of 30 or 30 rows of 1.
$30 = 2 \cdot 15$	There could be 2 rows of 15 or 15 rows of 2.
$30 = 3 \cdot 10$	There could be 3 rows of 10 or 10 rows of 3.
$30 = 5 \cdot 6$	There could be 5 rows of 6 or 6 rows of 5.
$30 = 6 \cdot 5$	The factors 5 and 6 are already listed.

There are 8 possible arrangements: 1 row of 30, 30 rows of 1, 2 rows of 15, 15 rows of 2, 3 rows of 10, 10 rows of 3, 5 rows of 6, or 6 rows of 5.

Key Vocabulary
factor pair, *p. 26*
prime factorization, *p. 26*
factor tree, *p. 26*

Study Tip
When making an organized list of factor pairs, stop finding pairs when the factors begin to repeat.

On Your Own

Now You're Ready
Exercises 8–15

List the factor pairs of the number.

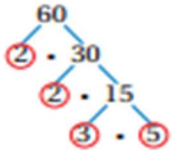
1. 18
2. 24
3. 51
4. **WHAT IF?** The woodwinds section of the marching band has 38 members. Which has more possible arrangements, the brass section or the woodwinds section? Explain.

Key Idea

Prime Factorization

The **prime factorization** of a composite number is the number written as a product of its prime factors.

You can use factor pairs and a **factor tree** to help find the prime factorization of a number. The factor tree is complete when only prime factors appear in the product. A factor tree for 60 is shown.



$60 = 2 \cdot 2 \cdot 3 \cdot 5$, or $2^2 \cdot 3 \cdot 5$

Remember
A *prime number* is a whole number greater than 1 with exactly two factors, 1 and itself. A *composite number* is a whole number greater than 1 with factors other than 1 and itself.

1.4 Exercises

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Vocabulary and Concept Check

1. **VOCABULARY** What is the prime factorization of a number?
2. **VOCABULARY** How can you use a factor tree to help you write the prime factorization of a number?
3. **WHICH ONE DOESN'T BELONG?** Which factor pair does not belong with the other three? Explain your reasoning.

2, 28	4, 14	6, 9	7, 8

Practice and Problem Solving

Use divisibility rules to determine whether the number is divisible by 2, 3, 5, 6, 9, and 10. Use a calculator to check your answer.

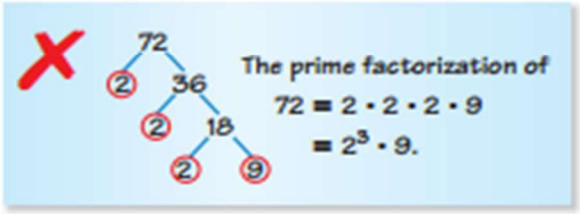
4. 1044
5. 1485
6. 1620
7. 1709

List the factor pairs of the number.

8. 15
9. 22
10. 34
11. 39
12. 45
13. 54
14. 59
15. 61

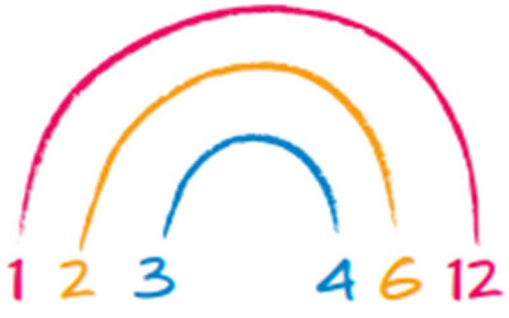
Write the prime factorization of the number.

16. 16
17. 25
18. 30
19. 26
20. 84
21. 54
22. 65
23. 77



24. **ERROR ANALYSIS** Describe and correct the error in writing the prime factorization.

25. **FACTOR RAINBOW** You can use a factor rainbow to check whether a list of factors is correct. To create a factor rainbow, list the factors of a number in order from least to greatest. Then draw arches that link the factor pairs. For perfect squares, there is no connecting arch in the middle. So, just circle the middle number. A factor rainbow for 12 is shown. Create factor rainbows for 6, 24, 36, and 48.



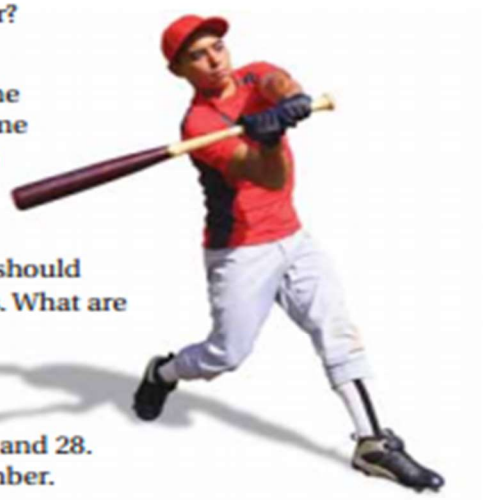
Find the number represented by the prime factorization.

26. $2^2 \cdot 3^2 \cdot 5$ **27.** $3^2 \cdot 5^2 \cdot 7$ **28.** $2^3 \cdot 11^2 \cdot 13$

Find the greatest perfect square that is a factor of the number.

29. 244 **30.** 650 **31.** 756 **32.** 1290

33. **CRITICAL THINKING** Is 2 the only even prime number? Explain.



34. **BASEBALL** The coach of a baseball team separates the players into groups for drills. Each group has the same number of players. Is the total number of players on the baseball team *prime* or *composite*? Explain.

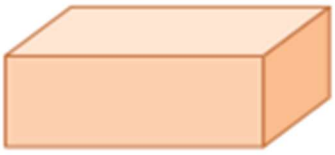
35. **SCAVENGER HUNT** A teacher divides 36 students into equal groups for a scavenger hunt. Each group should have at least 4 students but no more than 8 students. What are the possible group sizes?

36. **PERFECT NUMBERS** A *perfect number* is a number that equals the sum of its factors, not including itself. For example, the factors of 28 are 1, 2, 4, 7, 14, and 28. Because $1 + 2 + 4 + 7 + 14 = 28$, 28 is a perfect number. What are the perfect numbers between 1 and 28?

37. **BAKE SALE** One table at a bake sale has 75 cookies. Another table has 60 cupcakes. Which table allows for more rectangular arrangements when all the cookies and cupcakes are displayed? Explain.

38. **MODELING** The stage manager of a school play creates a rectangular acting area of 42 square yards. String lights will outline the acting area. To the nearest whole number, how many yards of string lights does the manager need to enclose this area?

Rectangular Prism



Volume = 40 cubic inches

39. **Volume** The volume of a rectangular prism can be found using the formula *volume* = *length* × *width* × *height*. Using only whole number dimensions, how many different prisms are possible? Explain.

Fair Game Review What you learned in previous grades & lessons

Find the difference. *(Skills Review Handbook)*

40. 192 − 47 **41.** 451 − 94 **42.** 3210 − 815 **43.** 4752 − 3504

44. **MULTIPLE CHOICE** You buy 168 pears. There are 28 pears in each bag. How many bags of pears do you buy? *(Skills Review Handbook)*

Ⓐ 5 Ⓑ 6 Ⓒ 7 Ⓓ 28

1.5 Lesson

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Factors that are shared by two or more numbers are called **common factors**. The greatest of the common factors is called the **greatest common factor** (GCF). One way to find the GCF of two or more numbers is by listing factors.

EXAMPLE 1 Finding the GCF Using Lists of Factors

Key Vocabulary
Venn diagram, *p. 30*
common factors, *p. 32*
greatest common factor, *p. 32*

Find the GCF of 24 and 40.

List the factors of each number.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

Circle the common factors.

The common factors of 24 and 40 are 1, 2, 4, and 8. The greatest of these common factors is 8.

∴ So, the GCF of 24 and 40 is 8.

Another way to find the GCF of two or more numbers is by using prime factors. The GCF is the product of the common prime factors of the numbers.

EXAMPLE 2 Finding the GCF Using Prime Factorizations

Find the GCF of 12 and 56.

Study Tip
Examples 1 and 2 show two different methods for finding the GCF. After solving with one method, you can use the other method to check your answer.

Make a factor tree for each number.

Write the prime factorization of each number.

$12 = 2 \cdot 2 \cdot 3$

$56 = 2 \cdot 2 \cdot 2 \cdot 7$

Circle the common prime factors.

$2 \cdot 2 = 4$

Find the product of the common prime factors.

∴ So, the GCF of 12 and 56 is 4.

On Your Own

Now You're Ready
Exercises 7–18

Find the GCF of the numbers using lists of factors.

1. 8, 36
2. 18, 72
3. 14, 28, 49

Find the GCF of the numbers using prime factorizations.

4. 20, 45
5. 32, 90
6. 45, 75, 120

EXAMPLE 3 Finding Two Numbers with a Given GCF

Which pair of numbers has a GCF of 15?

Ⓐ 10, 15 Ⓑ 30, 60 Ⓒ 21, 45 Ⓓ 45, 75

The number 15 cannot be a factor of the lesser number 10. So, you can eliminate Statement A.

The number 15 cannot be a factor of a number that does not have a 0 or 5 in the ones place. So, you can eliminate Statement C.

List the factors for Statements B and D. Then identify the GCF for each.

Choice B: **Factors of 30:** 1, 2, 3, 5, 6, 10, 15, 30

Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

The GCF of 30 and 60 is 30.

Choice D: **Factors of 45:** 1, 3, 5, 9, 15, 45

Factors of 75: 1, 3, 5, 15, 25, 75

The GCF of 45 and 75 is 15.

∴ The correct answer is Ⓓ.

EXAMPLE 4 Real-Life Application

You are filling piñatas for your sister's birthday party. The list shows the gifts you are putting into the piñatas. You want identical groups of gifts in each piñata with no gifts left over. What is the greatest number of piñatas you can make?

The GCF of the numbers of gifts represents the greatest number of identical groups of gifts you can make with no gifts left over. So, to find the number of piñatas, find the GCF.

$18 = 2 \cdot 3 \cdot 3$
$24 = 2 \cdot 3 \cdot 2 \cdot 2$
$42 = 2 \cdot 3 \cdot 7$

$2 \cdot 3 = 6$ Find the product of the common prime factors.

STEM VIDEO

The GCF of 18, 24, and 42 is 6.

∴ So, you can make at most 6 piñatas.

On Your Own

7. Write a pair of numbers whose greatest common factor is 10.

8. **WHAT IF?** In Example 4, you add 6 more pairs of earrings. Does this change your answer? Explain your reasoning.

Now You're Ready
Exercises 23–25

1.5 Exercises

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Vocabulary and Concept Check

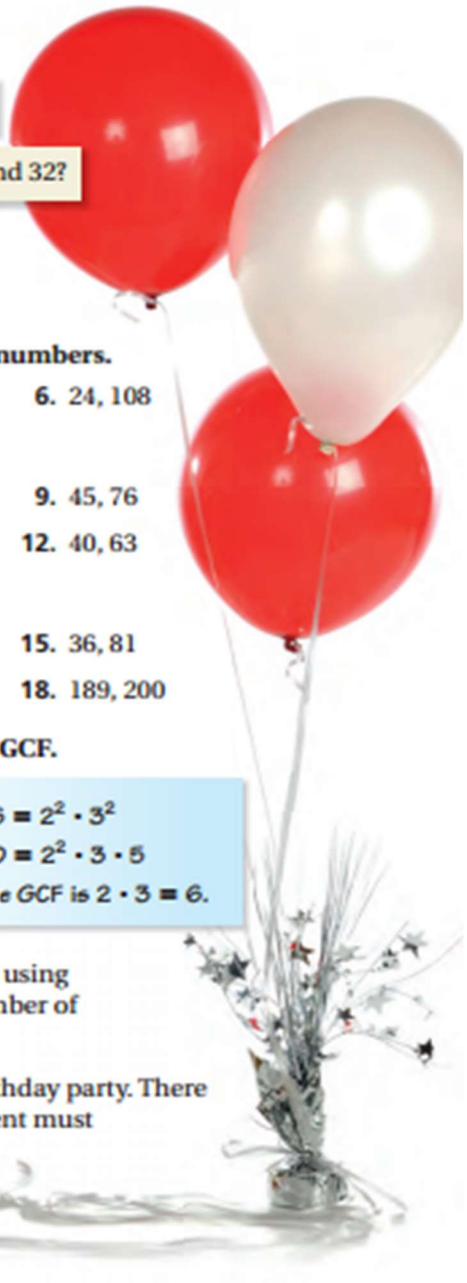
1. **VOCABULARY** What is the greatest common factor (GCF) of two numbers?
2. **WRITING** Describe how to find the GCF of two numbers by using prime factorization.
3. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What is the greatest common factor of 24 and 32?

What is the greatest common divisor of 24 and 32?

What is the greatest common prime factor of 24 and 32?

What is the product of the common prime factors of 24 and 32?



Practice and Problem Solving

Use a Venn diagram to find the greatest common factor of the numbers.

4. 12, 30
5. 32, 54
6. 24, 108

Find the GCF of the numbers using lists of factors.

1 7. 6, 15
8. 14, 84
9. 45, 76
10. 39, 65
11. 51, 85
12. 40, 63

Find the GCF of the numbers using prime factorizations.

2 13. 45, 60
14. 27, 63
15. 36, 81
16. 72, 84
17. 61, 73
18. 189, 200

ERROR ANALYSIS **Describe and correct the error in finding the GCF.**

19. ✗
$42 = 2 \cdot 3 \cdot 7$
$154 = 2 \cdot 7 \cdot 11$
The GCF is 7.

20. ✗
$36 = 2^2 \cdot 3^2$
$60 = 2^2 \cdot 3 \cdot 5$
The GCF is $2 \cdot 3 = 6$.

21. **CLASSROOM** A teacher is making identical activity packets using 92 crayons and 23 sheets of paper. What is the greatest number of packets the teacher can make with no items left over?

22. **BALLOONS** You are making balloon arrangements for a birthday party. There are 16 white balloons and 24 red balloons. Each arrangement must be identical. What is the greatest number of arrangements you can make using every balloon?

Find the GCF of the numbers.

23. 35, 56, 63 **24.** 30, 60, 78 **25.** 42, 70, 84

26. OPEN-ENDED Write a set of three numbers that have a GCF of 16. What procedure did you use to find your answer?

27. REASONING You need to find the GCF of 256 and 400. Would you rather list their factors or use their prime factorizations? Explain.

CRITICAL THINKING **Tell whether the statement is *always*, *sometimes*, or *never* true.**

28. The GCF of two even numbers is 2.

29. The GCF of two prime numbers is 1.

30. When one number is a multiple of another, the GCF of the numbers is the greater of the numbers.

31. BOUQUETS A florist is making identical bouquets using 72 red roses, 60 pink roses, and 48 yellow roses. What is the greatest number of bouquets that the florist can make if no roses are left over? How many of each color are in each bouquet?

32. VENN DIAGRAM Consider the numbers 252, 270, and 300.

a. Create a Venn diagram using the prime factors of the numbers.

b. Use the Venn diagram to find the GCF of 252, 270, and 300.

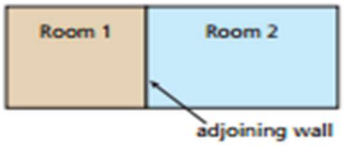
c. What is the GCF of 252 and 270? 252 and 300? Explain how you found your answer.

33. FRUIT BASKETS You are making fruit baskets using 54 apples, 36 oranges, and 73 bananas.

a. Explain why you cannot make identical fruit baskets without leftover fruit.

b. What is the greatest number of identical fruit baskets you can make with the least amount of fruit left over? Explain how you found your answer.

34. Problem Solving Two rectangular, adjacent rooms share a wall. One-foot-by-one-foot tiles cover the floor of each room. Describe how the greatest possible length of the adjoining wall is related to the total number of tiles in each room. Draw a diagram that represents one possibility.



Fair Game Review What you learned in previous grades & lessons

Tell which property is being illustrated. *(Skills Review Handbook)*

35. $13 + (29 + 7) = 13 + (7 + 29)$ **36.** $13 + (7 + 29) = (13 + 7) + 29$

37. $(6 \times 37) \times 5 = (37 \times 6) \times 5$ **38.** $(37 \times 6) \times 5 = 37 \times (6 \times 5)$

39. MULTIPLE CHOICE In what order should you perform the operations in the expression $4 \times 3 - 12 \div 2 + 5$? *(Section 1.3)*

Ⓐ $\times, -, \div, +$ Ⓑ $\times, \div, -, +$ Ⓒ $\times, \div, +, -$ Ⓓ $\times, +, -, \div$

1.6 Lesson

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Multiples that are shared by two or more numbers are called **common multiples**. The least of the common multiples is called the **least common multiple** (LCM). You can find the LCM of two or more numbers by listing multiples or using prime factors.

EXAMPLE 1 Finding the LCM Using Lists of Multiples

Key Vocabulary
common multiples, *p. 38*
least common multiple, *p. 38*

Find the LCM of 4 and 6.

List the multiples of each number.

Multiples of 4: 4, 8, (12), 16, 20, (24), 28, 32, (36), . . . Circle the common multiples.

Multiples of 6: 6, (12), 18, (24), 30, (36), . . .

Some common multiples of 4 and 6 are 12, 24, and 36. The least of these common multiples is 12.

∴ So, the LCM of 4 and 6 is 12.

On Your Own

Now You're Ready
Exercises 6–11

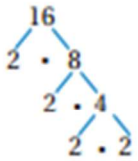
Find the LCM of the numbers using lists of multiples.

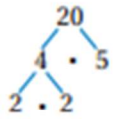
1. 3, 8
2. 9, 12
3. 6, 10

EXAMPLE 2 Finding the LCM Using Prime Factorizations

Find the LCM of 16 and 20.

Make a factor tree for each number.





Write the prime factorization of each number. Circle each different factor where it appears the greater number of times.

$16 = (2) \cdot (2) \cdot (2) \cdot (2)$ 2 appears more often here, so circle all 2s.

$20 = 2 \cdot 2 \cdot (5)$ 5 appears once. Do not circle the 2s again.

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 = 80$ Find the product of the circled factors.

∴ So, the LCM of 16 and 20 is 80.

On Your Own

Now You're Ready
Exercises 12–17

Find the LCM of the numbers using prime factorizations.

4. 14, 18
5. 28, 36
6. 24, 90

1.6 Exercises

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Vocabulary and Concept Check

1. **VOCABULARY** What is the least common multiple (LCM) of two numbers?
2. **WRITING** Describe how to find the LCM of two numbers by using prime factorization.

Practice and Problem Solving

Use a Venn diagram to find the least common multiple of the numbers.

3. 3, 7
4. 6, 8
5. 12, 15

Find the LCM of the numbers using lists of multiples.

6. 2, 9
7. 3, 4
8. 8, 9
9. 5, 8
10. 15, 20
11. 12, 18

Find the LCM of the numbers using prime factorizations.

12. 9, 21
13. 12, 27
14. 18, 45
15. 22, 33
16. 36, 60
17. 35, 50

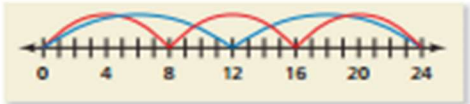
$6 \times 9 = 54$
The LCM of 6 and 9 is 54.

18. **ERROR ANALYSIS** Describe and correct the error in finding the LCM.

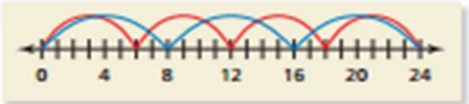
19. **AQUATICS** You have diving lessons every fifth day and swimming lessons every third day. Today you have both lessons. In how many days will you have both lessons on the same day again?
20. **HOT DOGS** Hot dogs come in packs of 10, while buns come in packs of eight. What are the least numbers of packs you should buy in order to have the same numbers of hot dogs and buns?
21. **MODELING** Which model represents an LCM that is different from the other three? Explain your reasoning.

A.

B.



C.



D.

Find the LCM of the numbers.

22. 2, 3, 7 **23.** 3, 5, 11 **24.** 4, 9, 12

25. 6, 8, 15 **26.** 7, 18, 21 **27.** 9, 10, 28

28. REASONING You need to find the LCM of 13 and 14. Would you rather list their multiples or use their prime factorizations? Explain.

CRITICAL THINKING Tell whether the statement is *always*, *sometimes*, or *never* true.

29. The LCM of two different prime numbers is their product.

30. The LCM of a set of numbers is equal to one of the numbers in the set.

31. The GCF of two different numbers is the LCM of the numbers.

32. SUBWAY At Union Station, you notice that three subway lines just arrived at the same time. The table shows their arrival schedule. How long must you wait until all three lines arrive at Union Station at the same time again?

Subway Line	Arrival Time
A	every 10 min
B	every 12 min
C	every 15 min

33. RADIO CONTEST A radio station gives away \$15 to every 15th caller, \$25 to every 25th caller, and free concert tickets to every 100th caller. When will the station first give away *all* three prizes to one caller?

34. TREADMILL You and a friend are running on treadmills. You run 0.5 mile every 3 minutes, and your friend runs 2 miles every 14 minutes. You both start and stop running at the same time and run a whole number of miles. What is the least possible number of miles you and your friend can run?

35. VENN DIAGRAM Refer to the Venn diagram.

a. Copy and complete the Venn diagram.

b. What is the LCM of 16, 24, and 40?

c. What is the LCM of 16 and 40? 24 and 40?

36. Number Sense When is the LCM of two numbers equal to their product?

Fair Game Review What you learned in previous grades & lessons

Write the product as a power. *(Section 1.2)*

37. 3×3 **38.** $5 \cdot 5 \cdot 5 \cdot 5$ **39.** $17 \times 17 \times 17 \times 17 \times 17$

40. MULTIPLE CHOICE Which two powers have the same value? *(Section 1.2)*

Ⓐ 1^3 and 3^1 Ⓑ 2^4 and 4^2 Ⓒ 3^2 and 2^3 Ⓓ 4^3 and 3^4

1.4–1.6 Quiz

Check It Out
Progress Check
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List the factor pairs of the number. *(Section 1.4)*

1. 48

2. 56

Write the prime factorization of the number. *(Section 1.4)*

3. 60

4. 72

Find the GCF of the numbers using lists of factors. *(Section 1.5)*

5. 18, 42

6. 24, 44, 52

Find the GCF of the numbers using prime factorizations. *(Section 1.5)*

7. 38, 68

8. 68, 76, 92

Find the LCM of the numbers using lists of multiples. *(Section 1.6)*

9. 8, 14

10. 3, 6, 16

Find the LCM of the numbers using prime factorizations. *(Section 1.6)*

11. 18, 30

12. 6, 24, 32

Add or subtract. Write the answer in simplest form. *(Section 1.6)*

13. $\frac{3}{5} + \frac{2}{3}$

14. $\frac{7}{8} - \frac{3}{4}$

15. PICNIC BASKETS You are creating identical picnic baskets using 30 sandwiches and 42 cookies. What is the greatest number of baskets that you can fill using all of the food? *(Section 1.5)*

16. RIBBON You have 52 inches of yellow ribbon and 64 inches of red ribbon. You want to cut the ribbons into pieces of equal length with no leftovers. What is the greatest length of the pieces that you can make? *(Section 1.5)*

17. MUSIC LESSONS You have piano lessons every fourth day and guitar lessons every sixth day. Today you have both lessons. In how many days will you have both lessons on the same day again? Explain. *(Section 1.6)*

18. HAMBURGERS Hamburgers come in packs of 20, while buns come in packs of 12. What is the least number of packs you should buy in order to have the same numbers of hamburgers and buns? *(Section 1.6)*