

Key Vocabulary

statistics, p. 392
 statistical question,
 p. 392

Statistics is the science of collecting, organizing, analyzing, and interpreting data. A **statistical question** is one for which you do not expect to get a single answer. Instead, you expect a variety of answers, and you are interested in the distribution and tendency of those answers.

Recall that a dot plot uses a number line to show the number of times each value in a data set occurs. Dot plots show the *spread* and the *distribution* of a data set.

EXAMPLE 1 Answering a Statistical Question

You conduct a science experiment on house mice. Your teacher asks you, “What is the weight of a mouse?”

a. Is this a statistical question? Explain.

- Because you can anticipate that the weights of mice will vary, it is a statistical question.

b. You weigh some mice and record the weights (in grams) in the table. Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.

Draw a number line that includes the least value, 18, and greatest value, 28. Then place a dot above the number line for each data value.

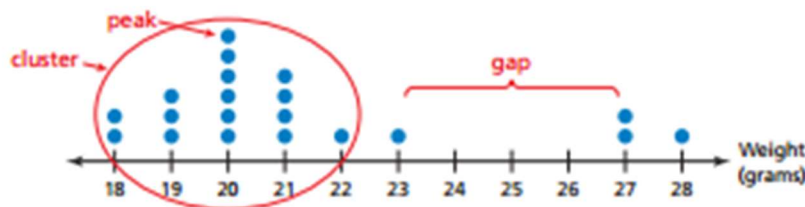


Weights (grams)

20	19	21	20
18	20	27	21
28	23	20	19
20	21	18	27
19	22	21	20

Study Tip

Dot plots are sometimes called *line plots*. It is easy to see clusters, peaks, and gaps in a dot plot.



Most of the data are clustered around 20. There is a peak at 20 and a gap between 23 and 27.

c. Use the distribution of the data to answer the question.

- Most mice weigh about 20 grams.

On Your Own

- The table shows the ages of some people who retired early. You are asked, “How old are people who retire early?”
 - Is this a statistical question? Explain.
 - Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.
 - Use the distribution of the data to answer the question.

Ages			
60	61	59	60
62	56	64	59
58	60	61	60
59	60	58	61

9.1 Exercises

Vocabulary and Concept Check

- VOCABULARY** What is a statistical question? Give an example.
- CRITICAL THINKING** What process can you use to answer a statistical question?
- NUMBER SENSE** The results of a survey are shown in the table. Did the survey ask a statistical question? Explain.

Miles			
6	1	9	2
2	5	4	9
8	10	6	6
5	1	8	1

Practice and Problem Solving

Answer the question. Tell whether your answer would be the same as your classmates'.

- How many inches are in 1 foot?
- How many pets do you have?
- On what day of the month were you born?
- How many senators are in Congress?

Determine whether the question is a statistical question. Explain.

- What is the eye color of sixth grade students?
- At what temperature (in degrees Fahrenheit) does water freeze?
- How many pages are in the favorite books of students your age?
- How many hours do sixth grade students use the Internet each week?

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.

12.

Number of Fouls					
2	1	2	0	0	2
2	1	6	1	1	0

13.

Camper Registrations				
21	25	25	22	21
23	24	26	25	16
24	26	22	25	22

14.

Years			
2011	2008	2013	2009
2009	2010	2010	2009
2010	2012	2009	2010

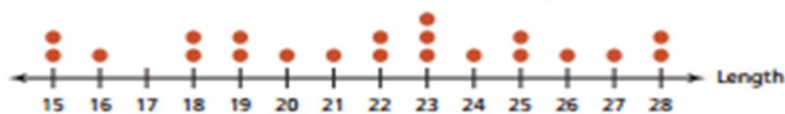
15.

Test Scores				
85	80	83	90	88
82	83	81	80	89
89	84	86	87	83

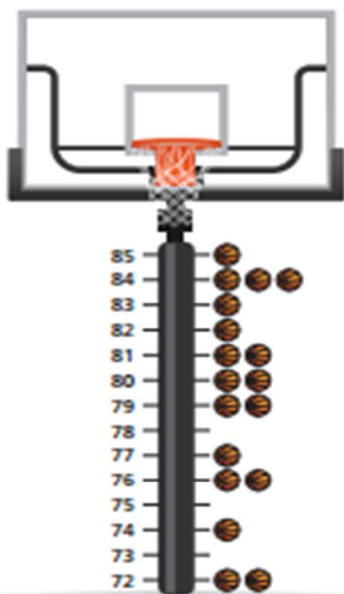
Hours of Homework			
2	4	3	2
1	2	2	1
2	3	5	2

16. **SURVEY** You conduct a survey to answer: "How many hours does a sixth grade student spend on homework during a school night?" The table shows the results.
- Is this a statistical question? Explain.
 - Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.
 - Use the distribution of the data to answer the question.

- 2 17. **EARTHWORMS** The dot plot shows the lengths of earthworms.



- How many earthworms does it represent?
 - How can you collect these data? What are the units?
 - Write a statistical question that you can answer using the dot plot. Then answer the question.
18. **BASKETBALL** The vertical dot plot shows the heights of the players on a recent NBA championship team.
- How many players were on the team?
 - How can you collect these data? What are the units?
 - Write a statistical question that you can answer using the dot plot. Then answer the question.



Use the Internet to research and identify the method of measurement and the units used when collecting data about the topic.

19. wind speed 20. amount of rainfall 21. earthquake intensity

The dot plot shows the speeds of cars in a traffic study. Estimate the speed limit. Explain your reasoning.

22.



23.



24. **REASONING** "How many letters are in the English alphabet?" is *not* a statistical question. Write a question about letters that is a statistical question. Explain your reasoning.
25. **Reasoning** A bar graph shows the favorite colors of 30 people. Does it make sense to describe the distribution of these data? Explain.



Fair Game Review

What you learned in previous grades & lessons

Tell whether the ordered pair is a solution of the equation. (Section 7.4)

26. $y = 4x$; (2, 8)

27. $y = 3x + 5$; (3, 15)

28. $y = 6x - 15$; (4, 9)

29. **MULTIPLE CHOICE** A point is reflected in the x -axis. The reflected point is (4, -3). What is the original point? (Section 6.5)

(A) (-3, 4)

(B) (-4, 3)

(C) (-4, -3)

(D) (4, 3)

9.2 Lesson

Key Vocabulary

mean, p. 398
outlier, p. 399

A mean is a type of average.

Key Idea

Mean

Words The **mean** of a data set is the sum of the data divided by the number of data values.

Numbers Data: 8, 5, 6, 9 Mean: $\frac{8 + 5 + 6 + 9}{4} = \frac{28}{4} = 7$
4 data values

EXAMPLE 1 Finding the Mean

Text Messages Sent

Mark: 120
Laura: 95
Stacy: 101
Josh: 125
Kevin: 82
Maria: 108
Manny: 90

The table shows the number of text messages sent by a group of friends over 1 week. What is the mean number of messages sent?

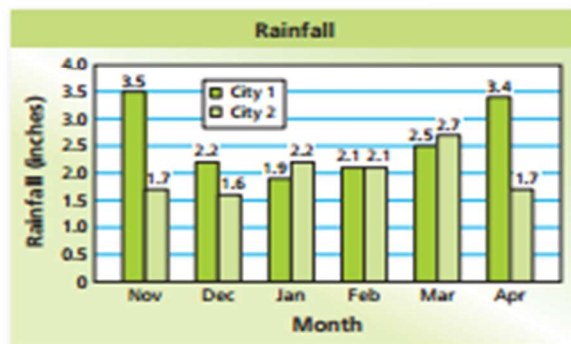
- (A) 100 (B) 102 (C) 103 (D) 104

$$\begin{aligned} \text{mean} &= \frac{120 + 95 + 101 + 125 + 82 + 108 + 90}{7} && \leftarrow \text{Sum of the data} \\ &= \frac{721}{7}, \text{ or } 103 && \leftarrow \text{Number of values} \\ & && \text{Simplify.} \end{aligned}$$

- ⚡ The mean number of text messages sent is 103. The correct answer is (C).

EXAMPLE 2 Comparing Means

The double bar graph shows the monthly rainfall amounts for two cities over a six-month period. Compare the mean monthly rainfalls.



$$\text{City 1 mean: } \frac{3.5 + 2.2 + 1.9 + 2.1 + 2.5 + 3.4}{6} = \frac{15.6}{6}, \text{ or } 2.6$$

$$\text{City 2 mean: } \frac{1.7 + 1.6 + 2.2 + 2.1 + 2.7 + 1.7}{6} = \frac{12}{6}, \text{ or } 2$$

- ⚡ Because 2.6 is greater than 2, City 1 averaged more rainfall.

On Your Own

Find the mean of the data.

- 49, 62, 52, 54, 61, 70, 55, 53
- 7.2, 8.5, 7.0, 8.1, 6.7

An **outlier** is a data value that is much greater or much less than the other values. When included in a data set, it can affect the mean.

EXAMPLE 3 Finding the Mean With and Without an Outlier

Shetland Pony Heights (inches)

40	37	39	40	42
38	38	37	28	40

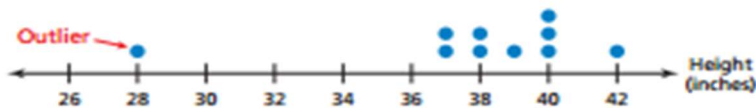
The table shows the heights of several Shetland ponies.

- Identify the outlier.
- Find the mean with and without the outlier.
- Describe how the outlier affects the mean.



STEM
VIDEO

- Display the data in a dot plot.



The height of 28 inches is much less than the other heights. So, it is an outlier.

- Mean with outlier:

$$\frac{40 + 37 + 39 + 40 + 42 + 38 + 38 + 37 + 28 + 40}{10} = \frac{379}{10}, \text{ or } 37.9$$

Mean without outlier:

$$\frac{40 + 37 + 39 + 40 + 42 + 38 + 38 + 37 + 40}{9} = \frac{351}{9}, \text{ or } 39$$

- With the outlier, the mean is less than all but three of the heights. Without the outlier, the mean better represents the heights.

On Your Own

For each data set, identify the outlier. Then describe how it affects the mean.

- Weights (in pounds) of dogs at a kennel:
48, 50, 55, 60, 8, 37, 50
- Prices for flights from Miami, Florida, to San Juan, Puerto Rico:
\$456, \$512, \$516, \$900, \$436, \$516



9.2 Exercises

Vocabulary and Concept Check

1. **VOCABULARY** Arrange the words to explain how to find a mean.

the data values

divide by

the number of data values

add

then

2. **NUMBER SENSE** Is the mean always equal to a value in the data set? Explain.
3. **REASONING** Can you use the mean to answer a statistical question? Explain.

Practice and Problem Solving

Describe an average value of the data.

4. Ages in a class: 11, 12, 12, 12, 12, 12, 13 5. Movies seen this week: 0, 0, 0, 1, 1, 2, 3

Find the mean of the data.

1

6.

Pets Owned	
Brandon	I
Jill	III
Mark	II
Nicole	III
Steve	0

7.

Brothers and Sisters	
Amanda	♂
Eve	♂ ♂ ♂ ♂ ♂
Joseph	♂ ♂ ♂ ♂
Michael	♂ ♂

8.

Sit-ups		
108	85	94
103	112	115
98	119	126
105	82	89

9.



10. **GOLF** The table shows tournament finishes for a golfer.

- a. What was the golfer's mean finish?
b. Identify two outliers for the data.

Tournament Finishes						
1	1	2	1	1	12	6
15	37	1	2	1	26	9

11. **COMMERCIALS** You and your friends are watching a television show. One of your friends asks, "How long are the commercial breaks during this show?"

- a. Is this a statistical question? Explain.
b. Use the mean of the values in the table to answer the question.

Time (minutes)				
4.2	3.5	4.55	2.75	2.25

Month	Rainfall (inches)	Month	Rainfall (inches)
Jan	2.22	Jul	3.27
Feb	1.51	Aug	5.40
Mar	1.86	Sep	5.45
Apr	2.06	Oct	4.34
May	3.48	Nov	2.64
Jun	4.57	Dec	2.14

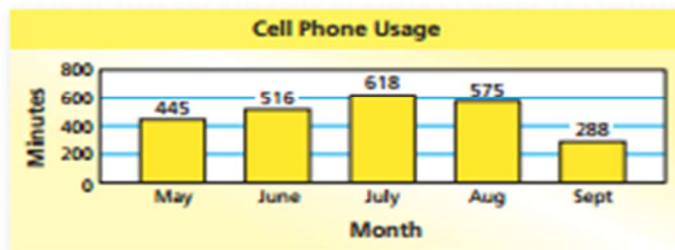
12. **RAINFALL** The table shows the monthly rainfall at a measuring station. What is the mean monthly rainfall?

13. **OPEN-ENDED** Create two different sets of data that have six values and a mean of 21.



14. **CELL PHONE** The bar graph shows your cell phone usage for five months.

- Which data value is an outlier? Explain.
- Find the mean with and without the outlier. Then describe how the outlier affects the mean.
- Describe a situation that could have caused the outlier in this problem.



15. **HEIGHT** The table shows the heights of the volleyball players from two schools. What is the difference between the mean heights of the two teams? Do outliers affect either mean? Explain.

	Player Height (inches)														
Dolphins	59	65	53	56	58	61	64	68	51	56	54	57			
Tigers	63	68	66	58	54	55	61	62	53	70	64	64	62	67	69

16. **REASONING** Make a dot plot of the data set 11, 13, 17, 15, 12, 18, and 12. Use the dot plot to explain how the mean is the point where the data set is balanced.
17. **ALLOWANCE** In your class, 7 students do not receive a weekly allowance, 5 students receive \$3, 7 students receive \$5, 3 students receive \$6, and 2 students receive \$8. What is the mean weekly allowance? Explain how you found your answer.
18. **Precision** A collection of 8 backpacks has a mean weight of 14 pounds. A different collection of 12 backpacks has a mean weight of 9 pounds. What is the mean weight of the 20 backpacks? Explain how you found your answer.



Fair Game Review

what you learned in previous grades & lessons

Evaluate the expression. (Section 1.3)

19. $\frac{8 + 10}{2}$

20. $\frac{26 + 34}{2}$

21. $\frac{18 + 19}{2}$

22. $\frac{14 + 17}{2}$

23. **MULTIPLE CHOICE** 60% of what number is 105? (Section 5.6)

(A) 63

(B) 175

(C) 630

(D) 1750

9.3 Lesson

Key Vocabulary

measure of center,
p. 404
median, p. 404
mode, p. 404

Study Tip

The mode is the only measure of center that you can use to describe a set of data that is not made up of numbers.

A **measure of center** is a measure that describes the typical value of a data set. The mean is one type of measure of center. Here are two others.

Key Ideas

Median

Words Order the data. For a set with an odd number of values, the **median** is the middle value. For a set with an even number of values, the **median** is the mean of the two middle values.

Numbers Data: 5, 8, 9, 12, 14 The median is 9.

Data: 2, 3, 5, 7, 10, 11

The median is $\frac{5+7}{2}$, or 6.

Mode

Words The **mode** of a data set is the value or values that occur most often. Data can have one mode, more than one mode, or no mode. When all values occur only once, there is no mode.

Numbers Data: 11, 13, 15, 15, 18, 21, 24, 24

The modes are 15 and 24.

EXAMPLE 1 Finding the Median and Mode

Bowling Scores

120	135	160	125	90
205	160	175	105	145

Find the median and mode of the bowling scores.

90, 105, 120, 125, 135, 145, 160, 160, 175, 205 Order the data.

Median: $\frac{135 + 145}{2} = \frac{280}{2}$, or 140 Add the two middle values and divide by 2.

Mode: 90, 105, 120, 125, 135, 145, 160, 160, 175, 205

The value 160 occurs most often.

∴ The median is 140. The mode is 160.

On Your Own

Find the median and mode of the data.

- 20, 4, 17, 8, 12, 9, 5, 20, 13
- 100, 75, 90, 80, 110, 102

Now You're Ready
Exercises 7–12

9.3 Exercises

Vocabulary and Concept Check

- NUMBER SENSE** Give an example of a data set that has no mode.
- WRITING** Which is affected most by an outlier: the mean, median, or mode? Explain.
- WHICH ONE DOESN'T BELONG** Which word does *not* belong with the other three? Explain.

median

outlier

mode

mean

- NUMBER SENSE** A data set has a mean of 7, a median of 5, and a mode of 8. Which of the numbers 7, 5, and 8 *must* be in the data set? Explain.

Practice and Problem Solving

Use grid paper to find the median of the data.

5. 9, 7, 2, 4, 3, 5, 9, 6, 8, 0, 3, 8

6. 16, 24, 13, 36, 22, 26, 22, 28, 25

Find the median and mode(s) of the data.

1 7. 3, 5, 7, 9, 11, 3, 8

8. 14, 19, 16, 13, 16, 14

9. 93, 81, 94, 71, 89, 92, 94, 99

10. 44, 13, 36, 52, 19, 27, 33

11. 12, 33, 18, 28, 29, 12, 17, 4, 2

12. 55, 44, 40, 55, 48, 44, 58, 67

13. **ERROR ANALYSIS** Describe and correct the error in finding the median of the data.



The median is 58.

63, 55, 49, 58, 50, 59, 51

Find the mode(s) of the data.

2 14.

Shirt Color		
Black	Blue	Red
Pink	Black	Black
Gray	Green	Blue
Blue	Blue	Red
Yellow	Blue	Blue
Black	Orange	Black
Black		

15.

Talent Show Acts		
Singing	Dancing	Comedy
Singing	Singing	Dancing
Juggling	Dancing	Singing
Singing	Poetry	Dancing
Comedy	Magic	Dancing
Poetry	Singing	Singing

16. **REASONING** In Exercises 14 and 15, can you find the mean and median of the data? Explain.

Find the mean, median, and mode(s) of the data. Choose the measure that best represents the data. Explain your reasoning.

- 3 17. 48, 12, 11, 45, 48, 48, 43, 32 18. 12, 13, 40, 95, 88, 7, 95
19. 2, 8, 10, 12, 56, 9, 5, 2, 4 20. 126, 62, 144, 81, 144, 103








Find the mean, median, and mode(s) of the data with and without the outlier. Describe the effect of the outlier on the measures of center.

- 4 21. 45, 52, 17, 63, 57, 42, 54, 58 22. 85, 77, 211, 88, 91, 84, 85

Find the mean, median, and mode(s) of the data.

23. 4.7, 8.51, 6.5, 7.42, 9.64, 7.2, 9.3 24. $8\frac{1}{2}$, $6\frac{5}{8}$, $3\frac{1}{8}$, $5\frac{3}{4}$, $6\frac{5}{8}$, $5\frac{1}{4}$, $10\frac{5}{8}$, $4\frac{1}{2}$

25. **WEATHER** The weather forecast for a week is shown.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
							
High	90° F	91° F	89° F	97° F	101° F	99° F	91° F
Low	74° F	78° F	77° F	77° F	83° F	78° F	72° F

- a. Find the mean, median, and mode(s) of the high temperatures. Which measure best represents the data? Explain your reasoning.
b. Repeat part (a) for the low temperatures.
26. **RESEARCH** Find the unit costs of 10 different kinds of cereal. Choose one cereal whose unit cost will be an outlier.
- a. Find the mean, median, and mode(s) of the data. Which measure best represents the data? Explain your reasoning.
b. Identify the outlier in the data set. Find the mean, median, and mode(s) of the data set without the outlier. Which measure does the outlier affect the most?



27. **PROBLEM SOLVING** The bar graph shows the numbers of hours you volunteered at an animal shelter. What is the minimum number of hours you need to work in the seventh week to justify that you worked an average of 10 hours for the 7 weeks? Explain your answer using measures of center.
28. **REASONING** Why do you think the mode is the least frequently used measure to describe a data set? Explain.

29. **MOTOCROSS** The ages of the racers in a bicycle motocross race are 14, 22, 20, 25, 26, 17, 21, 30, 27, 25, 14, and 29. The 30-year-old drops out of the race and is replaced with a 15-year-old. How are the mean, median, and mode of the ages affected?



30. **CAMERAS** The data are the prices of several digital cameras at a store.

\$130 \$170 \$230 \$130
\$250 \$275 \$130 \$185

- a. Does the price shown in the advertisement represent the prices well? Explain.
- b. Why might the store use this advertisement?
- c. In this situation, why might a person want to know the mean? the median? the mode? Explain.
31. **SALARIES** The table shows the monthly salaries for employees at a company.
- | Monthly Salaries (dollars) | | | | |
|----------------------------|------|------|------|------|
| 1940 | 1660 | 1860 | 2100 | 1720 |
| 1540 | 1760 | 1940 | 1820 | 1600 |
- a. Find the mean, median, and mode of the data.
- b. Each employee receives a 5% raise. Find the mean, median, and mode of the data with the raise. How does this increase affect the mean, median, and mode of the data?
- c. Use the original monthly salaries to calculate the annual salaries. Find the mean, median, and mode of the annual salaries. How are these values related to the mean, median, and mode of the monthly salaries?
32. **Critical Thinking** Consider the algebraic expressions $3x$, $9x$, $4x$, $23x$, $6x$, and $3x$. Assume $x > 0$.
- a. Find the mean, median, and mode.
- b. Is there an outlier? If so, what is it?



Fair Game Review

What you learned in previous grades & lessons

Find the value of the expression. (Section 1.1)

33. $48 - 35$ 34. $188 - 123$ 35. $416 - 297$ 36. $6249 - 3374$

37. **MULTIPLE CHOICE** A shelf in your room can hold at most 30 pounds. There are 12 pounds of books already on it. Which inequality represents the number of pounds you can add to the shelf? (Section 7.6)

(A) $x < 18$ (B) $x \geq 18$ (C) $x \leq 42$ (D) $x \leq 18$

9.1–9.3 Quiz

Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data. (Section 9.1)

1.

Weight (grams)			
42	40	37	42
43	41	42	43
37	41	41	42

2.

Time (seconds)				
63	66	65	60	59
59	64	58	65	58
64	60	59	64	63

Find the mean of the data. (Section 9.2)

3.

Tour Dates	
May	III
June	IIII IIII
July	IIII IIII IIII
August	IIII IIII IIII IIII
September	IIII

4.

Scores	
Judge 1	8.9
Judge 2	9.4
Judge 3	8.6
Judge 4	9.1

Find the median and the mode(s) of the data. (Section 9.3)

5. 3, 5, 9, 11, 3

6. 24, 4, 37, 56, 6, 56, 45

Find the mean, median, and mode(s) of the data. Choose the measure that best represents the data. Explain your reasoning. (Section 9.3)

7. 47, 147, 24, 47, 38, 42

8. 34, 57, 58, 56, 21

Hours of Exercise				
5	1	5	3	5
4	5	2	5	4
3	4	6	5	6

9. **EXERCISE** You conduct a survey to answer: "How many hours does a sixth-grade student spend exercising during a week?" The table shows the results. (Section 9.1)

a. Is this a statistical question? Explain.

b. Display the data in a dot plot. Identify any clusters, peaks, or gaps in the data.

c. Use the distribution of the data to answer the question.

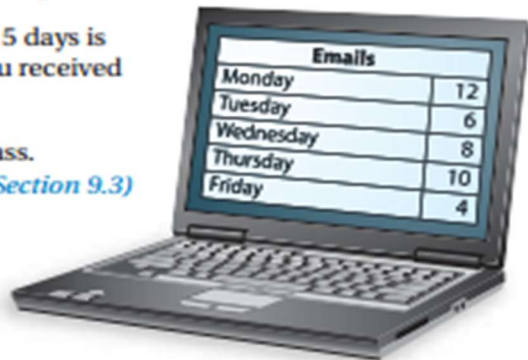
10. **EMAILS** The number of emails you received in 5 days is shown. What is the mean number of emails you received per day? (Section 9.2)

11. **QUIZZES** The data are your quiz scores for a class. Find the median and the mode of the data. (Section 9.3)

18, 19, 17, 14, 20, 20, 15, 21

12. **MUSIC** The data are the lengths of the songs (in minutes) on your new CD. Which measure of center best represents the data with and without the outlier? Explain. (Section 9.3)

2.2, 2.2, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 14.2



9.4 Lesson

A **measure of variation** is a measure that describes the distribution of a data set. A simple measure of variation to find is the **range**. The **range** of a data set is the difference between the greatest value and the least value.

EXAMPLE 1 Finding the Range

Key Vocabulary

measure of variation, p. 414
range, p. 414
quartiles, p. 414
first quartile, p. 414
third quartile, p. 414
interquartile range, p. 414

The table shows the lengths of several Burmese pythons captured for a study. Find and interpret the range of their lengths.

To find the least and the greatest values, order the lengths from least to greatest.

5, 6.25, 8, 10, 11, 12.5, 14, 15.5, 16.25, 18.5

The least value is 5. The greatest value is 18.5.

So, the range of the lengths is $18.5 - 5$, or 13.5 feet. This means that the lengths vary by no more than 13.5 feet.

Lengths (feet)	
18.5	8
11	10
14	15.5
12.5	6.25
16.25	5

On Your Own

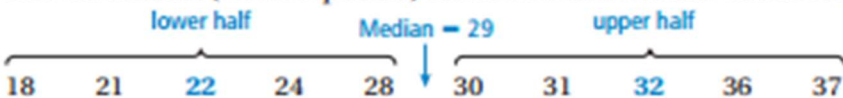
- The ages of people in line for a roller coaster are 15, 17, 21, 32, 41, 30, 25, 52, 16, 39, 11, and 24. Find and interpret the range of their ages.

Now You're Ready
Exercises 6–9

Key Ideas

Quartiles

The **quartiles** of a data set divide the data into four equal parts. Recall that the median (second quartile) divides the data set into two halves.



The median of the lower half is the **first quartile, Q_1** .

The median of the upper half is the **third quartile, Q_3** .

Interquartile Range (IQR)

The difference between the third quartile and the first quartile is called the **interquartile range**. The IQR represents the range of the middle half of the data and is another measure of variation.

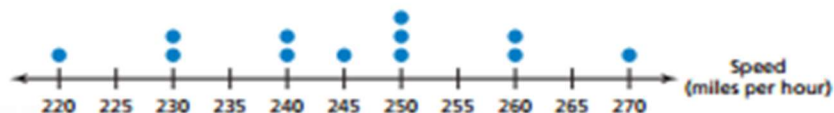
18	21	22	24	28	30	31	32	36	37
		IQR =	Q ₃	–	Q ₁				
		=	32	–	22				
		=	10						

Reading

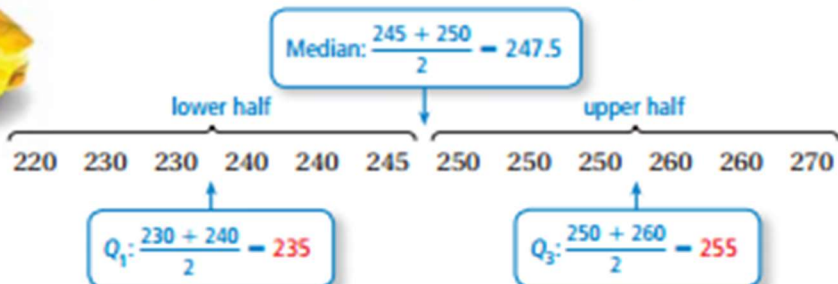
The first quartile can also be called the **lower quartile**. The third quartile can also be called the **upper quartile**.

EXAMPLE 2 Finding the Interquartile Range

The dot plot shows the top speeds of 12 sports cars. Find and interpret the interquartile range of the data.

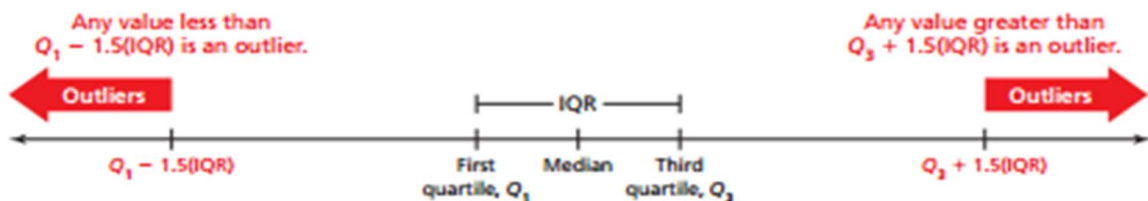


Order the speeds from slowest to fastest. Find the quartiles.



So, the interquartile range is $255 - 235 = 20$. This means that the middle half of the speeds vary by no more than 20 miles per hour.

You can use the quartiles and the interquartile range to check for outliers.

**EXAMPLE 3** Checking for Outliers

Check for outliers in the data set in Example 2.

$Q_1 - 1.5(\text{IQR})$	Outlier boundaries	$Q_3 + 1.5(\text{IQR})$
$235 - 1.5(20)$	Substitute values.	$255 + 1.5(20)$
205	Simplify.	285

There are no speeds less than 205 miles per hour or greater than 285 miles per hour. So, the data set has no outliers.

On Your Own

Now You're Ready
Exercises 11–14
and 17

2. The number of pages in each of an author's novels is shown.

356, 364, 390, 468, 400, 382, 376, 396, 350

- Find and interpret the interquartile range of the data.
- Does this data set contain any outliers? Justify your answer.

9.4 Exercises



Vocabulary and Concept Check

- VOCABULARY** How are measures of center different from measures of variation?
- VOCABULARY** How many quartiles does a data set have?
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

53, 47, 60, 45, 62, 59, 65, 50, 56, 48

What is the interquartile range of the data?

What is the range of the data?

What is the range of the middle half of the data?

What is the difference between the third quartile and the first quartile?

Practice and Problem Solving

Use grid paper to find the median of the data. Then find the median of the lower half and the median of the upper half of the data. Describe the spread of the data.

- 5, 8, 10, 1, 7, 6, 15, 8, 6
- 82, 62, 95, 81, 89, 51, 72, 56, 97, 98, 79, 85

Find the range of the data.

- 26, 21, 27, 33, 24, 29
- 52, 40, 49, 48, 62, 54, 44, 58, 39
- 133, 117, 152, 127, 168, 146, 174
- 4.8, 5.5, 4.2, 8.9, 3.4, 7.5, 1.6, 3.8
- ERROR ANALYSIS** Describe and correct the error in finding the range of the data.

 49, 48, 51, 41, 35, 44, 38
The range is $49 - 38$, or 11.

Find the median, first quartile, third quartile, and interquartile range of the data.

- 40, 33, 37, 54, 41, 34, 27, 39, 35
- 84, 75, 90, 87, 99, 91, 85, 88, 76, 92, 94
- 132, 127, 106, 140, 158, 135, 129, 138
- 38, 55, 61, 56, 46, 67, 59, 75, 65, 58

- PAPER AIRPLANE** The table shows the distances traveled by a paper airplane. Find and interpret the range and the interquartile range of the distances.

Distances (feet)			
$13\frac{1}{2}$	$21\frac{1}{2}$	21	$16\frac{3}{4}$
$10\frac{1}{4}$	19	32	$26\frac{1}{2}$
29	$16\frac{1}{4}$	$28\frac{1}{2}$	$18\frac{1}{2}$



16. **WRITING** Consider a data set that has no mode. Which measure of variation is greater, the range or the interquartile range? Explain your reasoning.
- 3 17. **OUTLIERS** Use the interquartile range to identify any outliers in Exercises 11–14.
18. **REASONING** How does an outlier affect the range of a data set? Explain.
19. **BASKETBALL** The table shows the numbers of points scored by players on a basketball team.

Points Scored					
21	53	74	82	84	93
103	108	116	122	193	

- a. Find the range and the interquartile range of the data.
- b. Use the interquartile range to identify the outlier(s) in the data set. Find the range and the interquartile range of the data set without the outlier(s). Which measure did the outlier(s) affect more?
20. **STRUCTURE** Two data sets have the same range. Can you assume that the interquartile ranges of the two data sets are about the same? Give an example to justify your answer.



21. **SINGING** The tables show the ages of the finalists for two reality singing competitions.

- a. Find the mean, median, range, and interquartile range of the ages for each show. Compare the results.
- b. A 21-year-old is voted off Show A, and the 36-year-old is voted off Show B. How do these changes affect the measures in part (a)? Explain.

Ages for Show A	
18	17
15	21
22	16
18	28
24	21

Ages for Show B	
21	20
23	13
15	18
17	22
36	25

22. **Open-Ended** Create a set of data with 7 values that has a mean of 30, a median of 26, a range of 50, and an interquartile range of 36.



Fair Game Review what you learned in previous grades & lessons

Find the mean of the data. (Section 9.2)

23. 8, 14, 22, 7, 2, 11, 25, 7, 5, 9
24. 55, 64, 58, 43, 49, 67

25. **MULTIPLE CHOICE** What is the surface area of the rectangular prism? (Section 8.2)

- (A) 62 m^2 (B) 72 m^2
 (C) 88 m^2 (D) 124 m^2



Key Vocabulary

mean absolute deviation, p. 420

Another measure of variation is the *mean absolute deviation*. The **mean absolute deviation** is an average of how much data values differ from the mean.

Key Idea

Finding the Mean Absolute Deviation (MAD)

Step 1: Find the mean of the data.

Step 2: Find the distance between each data value and the mean.

Step 3: Find the sum of the distances in Step 2.

Step 4: Divide the sum in Step 3 by the total number of data values.

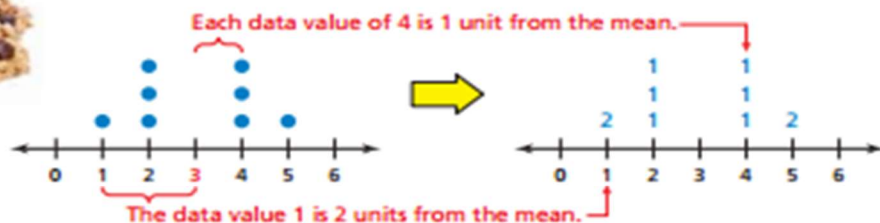
EXAMPLE 1 Finding the Mean Absolute Deviation

You record the numbers of raisins in 8 scoops of cereal. Find and interpret the mean absolute deviation of the data.

1, 2, 2, 2, 4, 4, 4, 5

Step 1: Mean = $\frac{1 + 2 + 2 + 2 + 4 + 4 + 4 + 5}{8} = \frac{24}{8} = 3$

Step 2: You can use a dot plot to organize the data. Replace each dot with its distance from the mean.



Step 3: The sum of the distances is $2 + 1 + 1 + 1 + 1 + 1 + 1 + 2 = 10$.

Step 4: The mean absolute deviation is $\frac{10}{8} = 1.25$.

So, the data values differ from the mean by an average of 1.25 raisins.

On Your Own

1. Find and interpret the mean absolute deviation of the data.

5, 8, 8, 10, 13, 14, 16, 22

Now You're Ready
Exercises 5–8

9.5 Exercises

Vocabulary and Concept Check

- REASONING** Describe a data set that has a mean absolute deviation of 0.
- WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.

range

interquartile range

mean

mean absolute deviation

Practice and Problem Solving

Find the average distance each data value in the set is from the mean. Round your answer to the nearest tenth, if necessary.

- Model years of used cars on a lot: 2010, 2002, 2005, 2007, 2001
- Prices of kites at a shop: \$7, \$20, \$9, \$35, \$12, \$15, \$7, \$10, \$20, \$25

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth, if necessary.

5.

Prices of Microphones (dollars)				
25	28	20	22	32
28	35	34	30	36

6.

Heights of 10-Year-Old Octuplets (inches)			
61	61	61	61
61	61	61	61

7.

Capacities of Stadiums (thousands of people)		
101.5	95.4	109.8
98.7	92.3	104.7

8.

Numbers of Visitors to a Website During a Week			
103	115	124	125
171	165	170	

9. **ERROR ANALYSIS** Describe and correct the error in finding the mean absolute deviation of the data set 35, 40, 38, 32, 42, and 41.



$$\text{mean} = \frac{35 + 40 + 38 + 32 + 42 + 41}{6} = 38$$

$$\text{MAD} = \frac{3 + 2 + 6 + 4 + 3}{5} = 3.6$$

So, the values differ from the mean by an average of 3.6.

10. **MUSEUMS** The data set shows the admission prices at several museums.

\$20, \$20, \$16, \$12, \$15, \$25, \$11

Find and interpret the range, interquartile range, and mean absolute deviation of the data.



11. **MENU** The table shows the prices of the five most-expensive and least-expensive dishes on a menu. Find the MAD of each data set. Then compare their variations.

Five Most-Expensive Dishes					Five Least-Expensive Dishes				
\$28	\$30	\$28	\$39	\$25	\$7	\$7	\$10	\$8	\$12

12. **COINS** The data sets show the years of the coins in two collections.
 Derek's collection: 1950, 1952, 1908, 1902, 1955, 1954, 1901, 1910
 Paul's collection: 1929, 1935, 1928, 1930, 1925, 1932, 1933, 1920



Find the measures of center and the measures of variation for each data set. Compare the measures. What can you conclude?

13. **PROBLEM SOLVING** You survey students in your class about the number of movies they watched last month. The results are shown in the table.

Movies Watched			
7	5	14	5
6	9	10	12
15	4	5	8
11	10	9	2

- a. Find the measures of center and the measures of variation for the data.
 b. A new student joins the class who watched 21 movies last month. Is 21 an outlier? How does including this value affect the measures of center and the measures of variation? Explain.

REASONING Which data set do you think would have the greater mean absolute deviation? Explain your reasoning.

14. guesses for number of gumballs in a jar
 guesses for number of baseballs in a jar
15. monthly rainfall amounts in a city
 monthly amounts of water used in a home
16. **REASONING** The MAD of a data set is considered a more reliable measure of variation than the range or the interquartile range. Why do you think this is true?
17. **Critical Thinking** Add and subtract the MAD from the mean in the original data set in Exercise 13.
 a. What percent of the values are within one MAD of the mean? two MADs of the mean? Which values are more than twice the MAD from the mean?
 b. What do you notice as you get more and more MADs away from the mean? Explain.



Fair Game Review

what you learned in previous grades & lessons

Find the mean, median, and mode(s) of the data. (Section 9.2 and Section 9.3)

18. 4, 6, 7, 9, 6, 4, 5, 6, 8, 10

19. 1.2, 1.7, 1.7, 2.1, 1.4, 1.2, 1.9

20. **MULTIPLE CHOICE** What is the surface area of the square pyramid? (Section 8.3)

- (A) 100.8 yd² (B) 147.2 yd²
 (C) 211.2 yd² (D) 368 yd²



9.4–9.5 Quiz



Find the range of the data. (Section 9.4)

1. 35, 76, 43, 58, 34, 67 2. 19, 21, 22, 22, 19, 25, 24, 23, 24

Find the median, first quartile, third quartile, and interquartile range of the data. (Section 9.4)

3. 56, 48, 72, 37, 35, 42, 48, 33, 28 4. 95, 14, 86, 62, 55, 46, 28, 37, 33, 70, 31

Find and interpret the mean absolute deviation of the data. Round your answer to the nearest tenth if necessary. (Section 9.5)

5.

Ages of Television Show Viewers (years)			
29	18	26	33
33	22	34	26

6.

Prices of Houses (thousands of dollars)				
80	120	95	240	140
75	135	110	90	125



7. **AMUSEMENT PARKS** The data set shows the admission prices at several amusement parks.

\$65, \$70, \$40, \$55, \$35, \$40, \$60

Find and interpret the range, interquartile range, and mean absolute deviation of the data. (Section 9.4 and Section 9.5)

8. **TEACHING EXPERIENCE** The tables show the years of teaching experience of faculty members at two schools. (Section 9.4)

- a. Find the mean, median, range, and interquartile range of the years of experience for each school. Compare the results.
- b. The teacher with 11 years of experience leaves School A, and the teacher with 33 years of experience retires from School B. How does this affect the measures in part (a)? Explain.

School A: Teaching Experience (years)		School B: Teaching Experience (years)	
5	11	4	15
10	22	6	12
7	8	10	33
8	6	12	20
10	35	8	7

9. **BOOK CLUB** You survey the students in your book club about the number of books they read last summer. The results are shown in the table. (Section 9.4 and Section 9.5)

Books Read			
8	14	15	9
6	12	9	13
11	11	7	5
12	6	10	8

- a. Find the measures of center and the measures of variation for the data.
- b. A new student who read 18 books last summer joins the club. Is 18 an outlier? How does adding this value to the data set affect the measures of center and variation? Explain.

